From Neugebauer and Sachs, *Mathematical Cuneiform Texts* (New Haven, 1945)

CHAPTER III. PROBLEM-TEXTS

§ 1. Introduction

As was stated in Chapter I, our knowledge with regard to the exact date and place of origin of our texts is so inadequate that we cannot expect to be able to arrange our material according to such viewpoints. What we can do is merely to separate the large group of Old-Babylonian tablets (ca. 1900 to 1600 B.C.) from the Neo-Assyrian (ca. 700 B.C.) and Neo-Babylonian and Seleucid (last six centuries B.C.) texts. Within the Old-Babylonian group, which consists of problem-texts A to Ue, we have arranged the tablets, so far as possible, according to subject matter. Problem-texts V to Y, of a later date, neither allow nor require special internal classification.

The Old-Babylonian texts vary greatly as to type and content. Some tablets contain only one example, which gives all the details of the solution of the problem stated at the beginning of the text. We even have tablets which contain only a part of the working out of the solution of a problem (C and Uc). At the other extreme are tablets which state hundreds of problems in a very condensed form but give no answers (e.g., T and U). Between these two extremes lie all sorts of intermediate types: texts with two or more examples which are worked out in detail and which are well arranged according to the degree of mathematical complication, texts with many examples of quite diverse character arranged very carelessly (this type is represented by the texts BM 85194 and BM 85210, both published in MKT I), and texts which represent a smaller collection of coordinated problems.

Several of the texts which present large numbers of problems without giving answers bear colophons giving the tablet a serial number. This gave rise to the name "Series Texts" used in MKT for this whole group of tablets. We think it wise, however, to abandon this name because the new material makes it difficult to define the borders of this group. It is worth stressing that there is no evidence for canonical mathematical "series" of the type of the astrological series "*Enūma Anu Enlil*" or the lexicographical "ḪAR-ra/*ḫubullu*." The same tablet number occurs two or more times on tablets with different contents; problem-text T, for instance, is called "Tablet 10" although the same number is given to another tablet.[98] On the other hand, T is clearly related to U, but U is not given a serial number. Thus, it is fairly clear that the numbering of these texts implies nothing more than an arrangement of tablets of various groups by a scribe to keep them in order.

The relationship between texts which state problems and merely give the answers and texts which give the details of the solution is nicely illustrated by problem-texts G, H and J. Problem-text G states 31 problems in a well planned order; the writing down of the details of the solution of all these problems required three tablets, the first and last of which are preserved in H and J. It is clear that the same might be assumed for many tablets with single problems and corresponding collections of examples.

The first two texts published below stand somewhat between the table-texts and the problem-texts. The first (§ 2) tabulates the answers to a problem containing Pythagorean numbers (or Pythagorean triangles). It is the oldest preserved document in ancient number theory. The second text (§ 3) deals with the finding of a cube root in a special case.

The next group of texts (§ 4) deals with geometrical problems. Most of the texts have to do with triangles and trapezoids and their subdivision according to given conditions. Although these problems are sometimes accompanied by figures (usually very much out of scale) and although their terminology is geometrical, the whole treatment is strongly algebraic. Geometry is involved only insofar as it leads to the necessary relations between the given quantities; the solution, however, proceeds in a purely algebraic manner, and we find nowhere any certain traces of the type of geometrical argument which is so familiar to us from the Greek treatment of problems of this sort.

Problems involving solid geometry (prism and truncated pyramid) are dealt with in texts characterized by the word ki-lá, which probably refers to some sort of excavation (§ 5). The texts which we collect under the heading "irrigation" (§ 6) are of a similarly practical character. Most of these examples have to do with the volume of earth excavated in the digging or enlarging of canals. The mathematical aspect is usually very simple, but we learn from these texts a great deal about wages, the number of workers employed, and metrological relations.

The greatest amount of new information about practical questions is yielded by texts dealing with bricks (§ 7). Here we learn for the first time the dimensions of several standard types of brick and the metrological system used in counting bricks. It is clear that such information is not only of use for the

[98] Cf. the list given MKT I p. 385.

better understanding of similar mathematical texts but will also influence our interpretation of economic documents and archeological data.

In § 8 we reach the most typical part of Babylonian mathematics, namely, problems of a predominantly algebraic character. The first three texts in this group contain short examples of various types: a simple inheritance problem (Q No. 2), linear equations resulting from the determination of the weight of a stone from certain given conditions (R), and quadratic equations for the sides of a rectangle (S). The last text is placed here because its geometrical element consists of nothing more than the fact that the area of a rectangle is given by the product xy of its sides. But this text leads to the problem of determining x and y from a given product xy and a sum $x + y$ or a given difference $x - y$. This is the normal form for quadratic equations, to which the more involved forms must be reduced. The great weight attributed to the solution of quadratic equations in Babylonian mathematics is evident from all our material. This is also shown by texts T and U, which contain 247 and 177 problems, respectively, all leading to quadratic equations. These texts are systematic compendia of exercises arranged according to a consistent scheme. The common principle which must be used in order to solve all these problems is similar to our use of an algebraic formula in which we are allowed to substitute special values for the letters a, b, c etc. which occur as coefficients.

An entirely new type of "mathematical" text is represented by Ud and Ue (§ 9). The tablets contain lists of numbers (to which are added short explanations) which occur in mathematical texts. We find here, e.g., coefficients referring to bricks, work assignment, etc.—in short, just those parameters which must be known by anyone dealing with various types of mathematical texts. For the first time we have texts which have the character of pages from a general handbook.

The later periods of Mesopotamian history are very meagrely represented in our material (§ 10). Only two problem-texts from the Seleucid period were published in MKT; we now add another text from the same period (Y) and two somewhat doubtful Late-Assyrian fragments (V and W). Between these late texts and the Old-Babylonian material there still remains a gap of about one thousand years, to which not a single problem-text can be assigned with certainty. It is worth noticing, however, that there is no basic deviation in character between the latest material and the Old-Babylonian.

§ 2. Pythagorean Numbers

A. Plimpton 322

(Photograph: Plate 25)

Obverse I	II	III	IV
1 *[ta-k]i-il-ti ṣi-li-ip-tim*	íb-si₈ sag	íb-si₈ *ṣi-li-ip-tim*	mu-bi-im
2 *[ša in-]na-as-sà-ḫu-ú-[m]a* sag *i-. . .-ú*			
3 [1,59],15	1,59	2,49	ki-1
4 [1,56,56],58,14,50,6[105],15	56,7	3,12,1[108]	ki-2
5 [1,55,7],41,15,33,45	1,16,41	1,50,49	ki-3
6 [1],5[3,1]0,29,32,52,16	3,31,49	5,9,1	ki-4
7 [1],48,54,1,40	1,5	1,37	ki[-5]
8 [1],47,6,41,40	5,19	8,1	[ki-6]
9 [1],43,11,56,28,26,40	38,11	59,1	ki-7
10 [1],41,33,59,3,45	13,19	20,49	ki-8
11 [1],38,33,36,36	9,1[106]	12,49	ki-9
12 [1],35,10,2,28,27,24,26,40	1,22,41	2,16,1	ki-10
13 [1],33,45	45	1,15	ki-11
14 [1],29,21,54,2,15	27,59	48,49	ki-12
15 [1],27,3,45	7,12,1[107]	4,49	ki-13
16 [1],25,48,51,35,6,40	29,31	53,49	ki-14
17 [1],23,13,46,4[0]	56	53[109]	ki[-15]

105 50,6 written like 56.

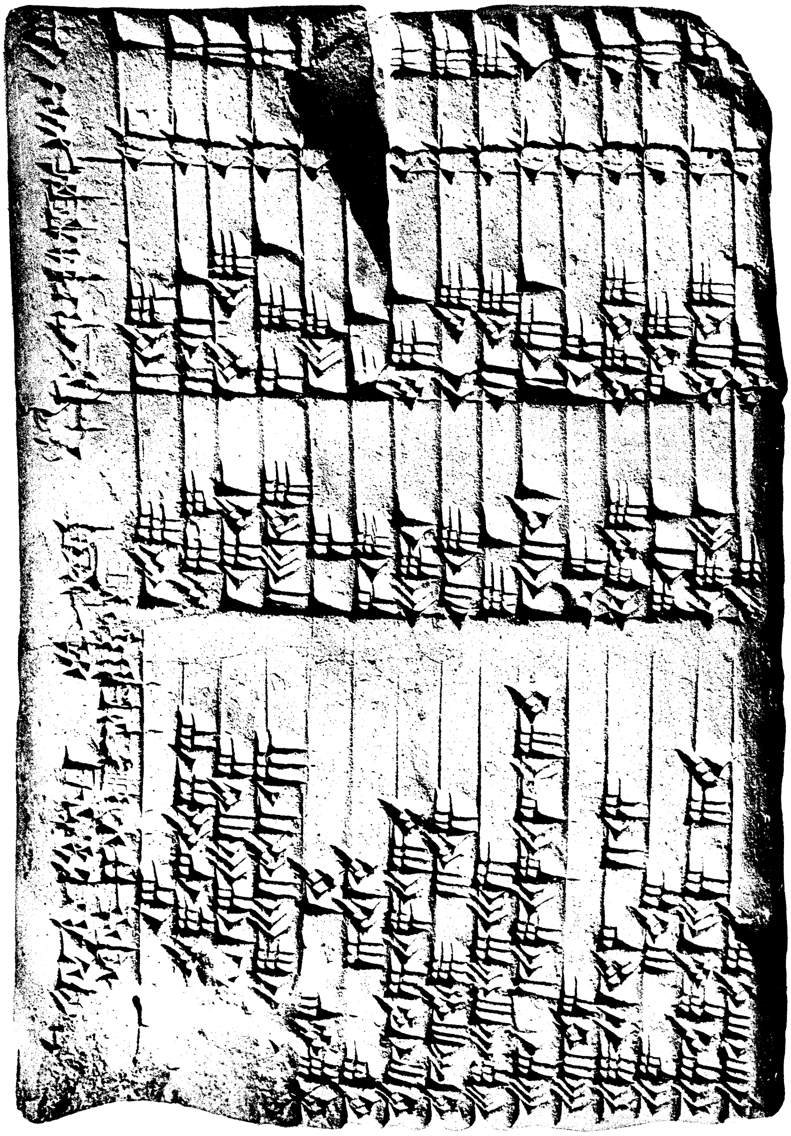
106 9,1 error for 8,1.

107 7,12,1 (the square of 2,41) error for 2,41.

108 3,12,1 error for 1,20,25.

109 53 error for 1,46 (i.e., $2 \cdot 53$); cf. below pp. 40 and 41.

A



Plimpton 322

Commentary

a. Description of the Tablet

In its present state, the tablet represents the right-hand part of a larger text. The presence of modern glue, until the recent baking of the tablet, on the left (broken) edge shows that the missing part must have been lost after the excavation of the tablet. The size of the preserved part, $4\frac{7}{8}$ by $3\frac{3}{8}$ in. (12.7 by 8.8 cm), would make it unlikely that much more than half (or even less) of the existing part is missing. The reverse is uninscribed.

The script is clearly Old-Babylonian, i.e., it falls in the period between 1900 and 1600 B.C. The sign for 9 consists of three superimposed rows of three vertical wedges each. Zeros are not indicated by a special sign, but a blank space occurs in lines 3 and 15 where zero is called for; on the other hand, lines 7, 8, 10, etc., show that a blank space does not necessarily indicate zero.

Nothing is known concerning the provenance of the tablet. [The sixth plate in Mendelsohn, Catalogue, which was published months after this section was written, contains a photograph of the tablet before it was baked. The photo which we publish on Plate 22 is of the baked and cleaned tablet. The content of the tablet is characterized as "commercial account" by Mendelsohn.]

b. Content

The text deals with "Pythagorean triangles": right triangles whose sides are integers. Let l denote the longer, b the shorter leg of a right triangle, d its hypotenuse; then l, b and d are integers which fulfill the relation

$$(1) \qquad l^2 + b^2 = d^2.$$

The values of d and b for 15 such Pythagorean triangles are given in the second and third preserved columns of our text. One might assume that the missing part contained the corresponding values of l. The first preserved column gives the ratios of d^2 to l^2. Although the values of d and b vary in a very irregular manner, the ratios $\frac{d^2}{l^2}$ decrease almost linearly (cf. fig. 2). Because the difference from line to line is very small (average 0;2,34), this is virtually equivalent to saying that $\frac{d}{l}$ decreases almost linearly (average difference 0;0,59,17,$\cdots$ i.e., almost 0;1), and we shall see that this is indeed the proper formulation of the problem. Formulating the problem with respect to the triangles, we can say that we start out with almost half a square (because the value of $b:l$ which corresponds to the first line [line 3] is 0;59,30) and gradually diminish the angle between l and d step by step, the

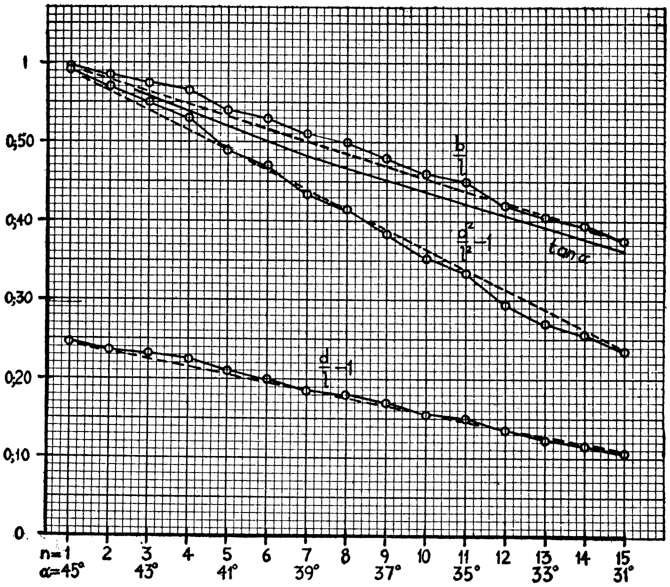


FIG. 2.

lowest value being almost exactly 31°. It must, however, be kept in mind that the actual size of these triangles varies considerably owing to the fact that all sides are integral solutions of (1) and not approximations.

The numbers 1, 2, $\cdots$, 15 in the last column have no exact relationship to the preceding numbers but merely indicate the number of the steps, like the units on the abscissa in our diagram. The preceding ki gives them the character of ordinal numbers.[110]

c. The Headings

Each column contains a heading describing the contents. The fourth column has mu-bi-im, "its name," a translation which goes well with the interpretation of the numbers 1 to 15 made above.

The columns for b and d are headed íb-si$_8$ sag, "íb-si$_8$ of the width," and íb-si$_8$ *ṣi-li-ip-tim*, "íb-si$_8$ of the diagonal," respectively. The íb-si$_8$ is difficult to translate because of its unprecise character, or, more accurately stated, because of the lack of a corresponding expression in our terminology. The most common meaning is "side of a square" or "square root."[111] More generally, however, íb-si$_8$ seems to indicate the number which is the result of some operation or the number solving a certain problem.[112] In the present case, íb-si$_8$ sag (or *ṣi-li-ip-tim*) means something like "solving number of the width (or the diagonal)."

Two lines are given at the top of the first column but both are unfortunately damaged at the beginning.

[110] This use of ki is also known from other mathematical texts; cf., e.g., MKT I p. 248 and the Vocabulary below, s.v.

[111] See the instances cited in the Vocabulary under si$_8$.

[112] Cf. above p. 36 and note 96c.

The restoration given below seems probable although we are unable to read the last word of the second line.

[ta-k]i-il-ti ṣi-li-ip-tim
[ša in-]na-as-sà-ḫu-ú-[m]a sag *i-· · · -ú*

The translation causes serious difficulties. The most plausible rendering seems to be: "The *takiltum* of the diagonal which has been subtracted such that the width. . . ." The reading *[ta-k]i-il-ti* is, however, not certain; in addition, we do not know how it is to be translated here.[113] The mention of a subtraction and of the width in the second line could indicate that l^2 in $\frac{d^2}{l^2}$ is replaced by $d^2 - b^2$. This would be natural if l were given in the missing first column and if d and b were considered the unknown quantities to be determined which satisfy (1). We are, however, unable to give a sensible translation of this passage leading to $\frac{d^2}{d^2 - b^2}$.

d. Method of Solution

We turn now to the question of great historical interest: How were the mathematicians of the Old-Babylonian period able not only to solve the Pythagorean equation (1) in integers but to adapt the solutions to the further condition that the proportion $\frac{d}{l}$ decrease from step to step by a number deviating very little from one-sixtieth?

In order to answer this question, we must first compare the solutions for l, b, and d given in the tablet. These are listed in the following table with the correction of the four errors in lines 4, 11, 15, and 17 mentioned in the foot-notes to the transcription:

Line	l	b	d
3	2, 0	1,59	2,49
4	57,36	56, 7	1,20,25
5	1,20, 0	1,16,41	1,50,49
6	3,45, 0	3,31,49	5, 9, 1
7	1,12	1, 5	1,37
8	6, 0	5,19	8, 1
9	45, 0	38,11	59, 1
10	16, 0	13,19	20,49
11	10, 0	8, 1	12,49
12	1,48, 0	1,22,41	2,16, 1
13	1, 0	45	1,15
14	40, 0	27,59	48,49
15	4, 0	2,41	4,49
16	45, 0	29,31	53,49
17	1,30	56	1,46

[113] See the discussion p. 130.

Before proceeding, it is necessary to note in the first place that line 13 gives the 15-fold values of the triangle with the sides $l = 4$, $b = 3$, $d = 5$; and secondly, that the last line contains the common factor 2. All the other solutions are relatively prime.

The most important feature in this table is the obvious difference in character of the numbers l on the one hand and the b's and d's on the other: the b's and d's are "complicated" numbers, but the l's are very "simple." This offhand impression can be translated into precise terms by using the well known theorem[114] that all relatively prime Pythagorean numbers are contained exactly once in the set of numbers

$$(2) \qquad l = 2pq \qquad b = p^2 - q^2 \qquad d = p^2 + q^2$$

where p and q are relatively prime integers, both not being simultaneously odd and $p > q$. If one calculates the numbers p and q which bring our numbers l, b, and d to the form (2), one will find that the p's as well as the q's are "regular numbers." In other words, the p's and q's can be characterized by three numbers α, β, and γ, the exponents of 2, 3, and 5, respectively. If we write[115]

$$2^\alpha 3^\beta 5^\gamma = (\alpha, \beta, \gamma),$$

we then obtain the following list of numbers p and q which satisfy (2):[116]

Line	p	q
3	12 = (2,1,0)	5 = (0,0,1)
4	1, 4 = (6,0,0)	27 = (0,3,0)
5	1,15 = (0,1,2)	32 = (5,0,0)
6	2, 5 = (0,0,3)	54 = (1,3,0)
7	9 = (0,2,0)	4 = (2,0,0)
8	20 = (2,0,1)	9 = (0,2,0)
9	54 = (1,3,0)	25 = (0,0,2)
10	32 = (5,0,0)	15 = (0,1,1)
11	25 = (0,0,2)	12 = (2,1,0)
12	1,21 = (0,4,0)	40 = (3,0,1)
13	2 = (1,0,0)	1 = (0,0,0)
14	48 = (4,1,0)	25 = (0,0,2)
15	15 = (0,1,1)	8 = (3,0,0)
16	50 = (1,0,2)	27 = (0,3,0)
17	9 = (0,2,0)	5 = (0,0,1)

The sense in which the numbers $l = 2pq$ are "simple" is now made clear by this list: they are numbers of the form (α, β, γ), i.e., so-called "regular

[114] Kronecker, Zahlentheorie p. 31.
[115] As above, p. 15.
[116] We include here the values from lines 13 and 17 although a common factor 15 of l, b, and d occurs in line 13, and p and q are both odd in line 17.

numbers" or numbers whose reciprocals are *finite* sexagesimal fractions.[117]

This latter quality of the p's and q's also yields the answer to the question how the numbers of our list were found which not only solve equation (1) but also furnish within narrow limits given proportions $\frac{d}{l}$. Using (2), we find for this proportion

$$(3) \qquad \frac{d}{l} = \tfrac{1}{2}(p\cdot\bar{q} + q\cdot\bar{p})$$

where $\bar{p}$ and $\bar{q}$ represent the reciprocals of p and q, respectively. In other words, our problem *requires* that p and q be regular numbers in order to obtain expressions (3) with finite sexagesimal fractions.

We can say more. With the single exception of $2{,}5 = 5^3$, all the numbers p and q of our list are contained in the group of regular numbers which constitute the "reciprocal tables."[118] As we have already mentioned,[119] the complete system of "multiplication tables" (represented in the "combined multiplication tables"[120]) is not a system of all products $a\cdot b$ ($1 \leqq a < 60$, $1 \leqq b < 60$) but only of products $a\bar{b}$ where $\bar{b}$ is the reciprocal of a regular number included in the reciprocal table.

Our final result, then, is that our tablet was calculated by selecting numbers $p\bar{q}$ and $q\bar{p}$ from combined multiplication tables such that (3) has a value as near as possible to the required values of $\frac{d}{l}$; Pythagorean numbers were then formed with these values of p and q according to (2).

Several remarks remain to be made. The exception of 2,5 mentioned above is not to be considered serious because we know that the usual reciprocal tables were occasionally enlarged in this very direction.[121] Secondly, instead of using (3), one can also produce Pythagorean numbers by using one parameter α and its reciprocal $\bar{\alpha}$ where $\alpha = \frac{p}{q}$.[122] But a comparison of the following four lines

p	q	α	$\bar{\alpha}$
12	5	2;24	0;25
1,4	27	2;22,13,20	0;25,18,45
1,15	32	2;20,37,30	0;25,36
2,5	54	2;18,53,20	0;25,55,12

shows immediately that neither α nor $\bar{\alpha}$ can have been the point of departure, but only the simple numbers p and q; hence formula (3) must necessarily have been used. Finally, the values $d = 53$ and $b = 56$ in the last line are obviously incorrect because this would imply $d < b$. The correction adopted in the preceding discussion assumes the triplet $d = 1{,}46$ (twice the value given in the text), $b = 56$, and $l = 1{,}30$. If, however, we keep $d = 53$ but assume $b = 28$ (half the value given in the text), we would also get Pythagorean numbers ($l = 45$). In this case, however, b, and not l, should be represented as $2pq$ and $p = 2$, $q = 7$ would be the corresponding numbers. Because $\frac{1{,}46}{1{,}30} = \frac{53}{45}$, the value of $\frac{d}{l}$ $\left(\text{and thus of } \frac{d^2}{l^2}\right)$ is the same in both cases and in agreement with the value given in Col. I. Only the first triplet, however, fits in with the method followed in the preceding lines.

e. Historical Consequences

The final remarks about the character of Babylonian mathematics in MKT III contain the sentence:[123] "Man wird also erwarten können, dass noch eine Art elementarer Zahlentheorie erkennbar wird—etwa so, dass das "pythagoreisch" der älteren historischen Schule besser "babylonisch" wird heissen dürfen." This is fully confirmed by the text discussed here. We now have a text of purely number theoretical character, treating a problem organically developed from other problems already well known and solved by using exactly those tools the development of which is so characteristic for Babylonian numerical methods.

We now see that Babylonian number theory was acquainted with rules like (2) to produce Pythagorean numbers, i.e., a theorem like Euclid X 29 lemma 1.

There can be little doubt that the Pythagorean numbers did not remain the only problem treated by this part of Babylonian mathematics. We have an explicit hint in this direction from the extant material itself: tables giving the powers c^n for exponents $n = 1, 2, \ldots, 10$ for the bases $c = 9, 16, 1{,}40$ and $3{,}45$.[124] All these are regular numbers, and it would be only natural to extend both problems and methods as described above to combinations of other numbers and different exponents. The study of sequences Σn, Σn^2 etc.[125] points in the same direction. Details can be disclosed only by the discovery of new texts, but their general direction seems evident.

In summary, our text gives the final link which connects the different parts of Old-Babylonian mathematics by the investigation of the fundamental laws of the numbers themselves.

[117] Cf., e.g., Neugebauer, Vorlesungen pp. 6 and 12ff.; and above p. 15.
[118] Cf. p. 11.
[119] P. 11.
[120] Pp. 24ff.
[121] Cf. pp. 13ff. above and MKT I pp. 23f. (Nos. 3 and 4).
[122] Cf. Dickson, History II, pp. 163ff.
[123] MKT III p. 80.
[124] MKT I pp. 77ff. and p. 35 above.
[125] MKT I pp. 102f., 497f., and MKT III pp. 13f.

§ 3. Cube Root

Aa. YBC 6295

(Photograph: Plate 46; copy: Plate 22)

Transcription

Obverse

1 [*ma-*]*ak-ṣa-ru-um ša* ba-si
2 ba-si 3,22,30 en-nam
3 *aš-šum* 3,22,30 ba-si *la id-di-nu-kum*
4 7,30 *ša* ba-si *i-na-di-nu-kum*
5 *ša-pa-al* 3,22,30 gar-ra-ma
6 3,22,30 7,30
7 ba-si 7,30 en-nam 30
8 igi 7,30 *pu-ṭur-ma* 8
9 8 *a-na* <3,>22,30 ÍL 27
10 ba-si 27 en-nam 3
11 3 ba-si *a-na* 30 ba-si *ša-ni-im*
12 ÍL 1,30
13 ba-si 3,22,30-e 1,30

Reverse

(Top and most of left edge broken; remainder uninscribed except for *a-na* ^{d}*sin*(EN-ZU) written about two-thirds of the way down, starting at about halfway across the width.)

Translation

Obverse

1 [*M*]*akṣarum*[125a] of the cube root.
2 What is the cube root of 3,22,30?
3 Since, (as regards) 3,22,30, they did not give you the cube root,
4–5 put (down) 7,30,0, the cube root of which they do give you, below 3,22,30:
6 3,22,30 7,30,0[125b].
7 What is the cube root of 7,30,0? (The answer is) 30.
8 Take the reciprocal of 7,30,0, and (the result is) 0;0,0,8.
9 Multiply 0;0,0,8 by 3,22,30, (and the result is) 27.
10 What is the cube root of 27? (The answer is) 3.
11–12 Multiply 3, the cube root, by 30, the other cube root,(and the result is) 1,30.
13 The cube root of 3,22,30 is 1,30.

[125a] See p. 55, note 152 for an inconclusive discussion of this word. The meaning here is possibly something like "model example."

[125b] The use of *šapal*, "below," in line 5 to indicate the relative position of the numbers in line 6 is apparently further proof that Old-Babylonian tablets were written and meant to be read turned 90° clockwise with respect to the position conventionally given by Assyriologists. For evidence from the figures in mathematical texts, cf. below, p. 49, note 135d and Neugebauer, Vorlesungen, pp. 34 and 176.

Reverse

(See transcription.)

Commentary

The cube root of a given number $b = 3,22,30$ is to be found. The solution of the problem presented in the text can be followed with ease. The text proceeds toward the solution by introducing an auxiliary number $a = 7,30,0$, of which the cube root $\sqrt[3]{7,30,0} = 30$ is said to be "given" (presumably in a table-text). The cube root of b is then calculated by

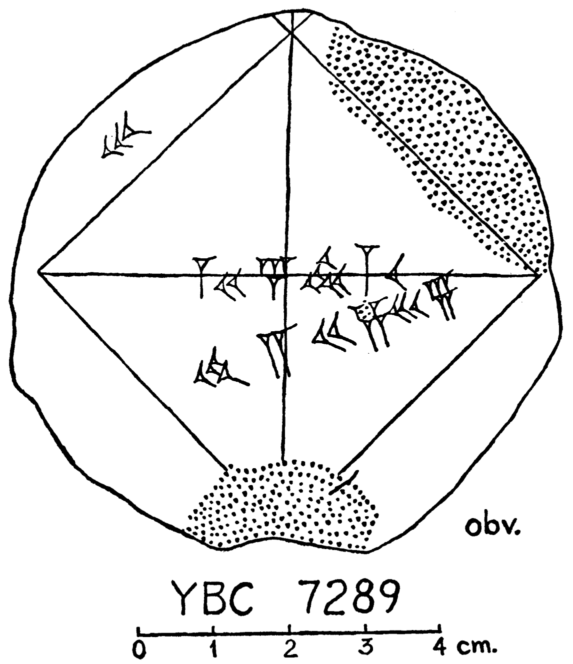
$$\sqrt[3]{b} = \sqrt[3]{a} \cdot \sqrt[3]{\frac{b}{a}} = 1,30.$$

For this method of solution, it is necessary that the auxiliary number a satisfy the following three conditions: (1) that a be the cube of a rational number; (2) that a be a regular number; (3) that $\sqrt[3]{\frac{b}{a}}$ can be found.

§ 4. Geometrical Problems

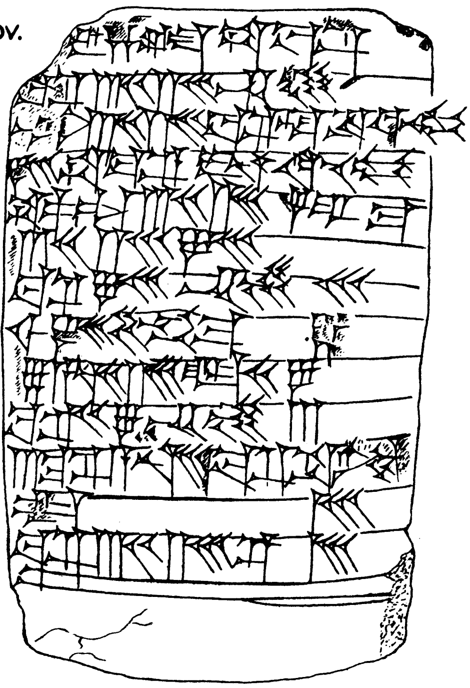
Simple Problems

a. Diagonal of a Square



YBC 7289, like the four texts which follow, is apparently Old-Babylonian. The obverse gives the fol-

Obv.



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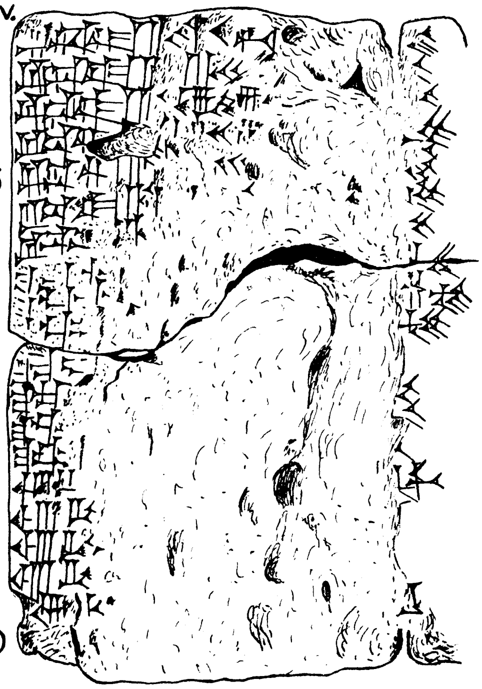

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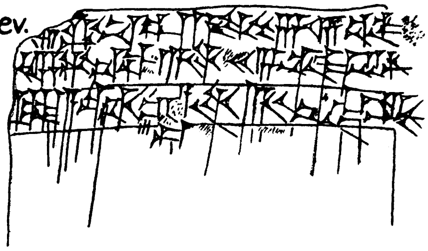

Sa. YBC 6492

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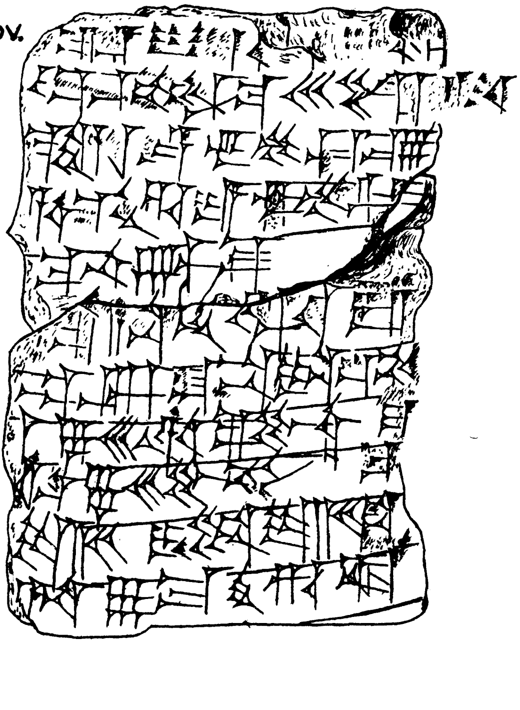

Rev.

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MLC 1842

lowing figure:[125c]

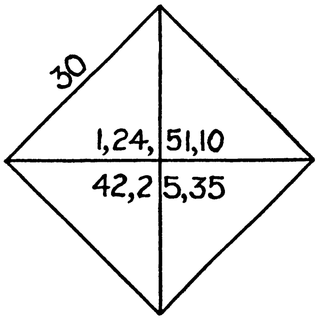


FIG. 3.

The number 30 indicates the side a of the square, and 1,24,51,10 means

(1) $$1;24,51,10 \approx \sqrt{2};$$

we therefore find

$$d = a\sqrt{2} = 42;25,35$$

for the diagonal. The value (1) for $\sqrt{2}$ is very good, as can be seen from

$$1;24,51,10^2 = 1;59,59,59,38,1,40.$$

Only the approximation $\sqrt{2} \approx 1;25$ occurred in previously published material (from the Seleucid period).[125d] Our new value, however, now also occurs in an Old-Babylonian list of coefficients, published p. 136 below, where we find in line 10 the entry

1,24,51,10 Diagonal, square root.

The question naturally arises how the value (1) for $\sqrt{2}$ was obtained. The following might give the answer, but it is impossible to furnish direct proof that precisely this way was followed. Two factors speak in favor of this explanation: first, that the process indicated leads to the very number found in (1); and, secondly, that the same procedure is attested in another text in finding the approximate value of $\sqrt{28,20}$.[125e]

The procedure in question consists in the alternating approximation of $\sqrt{a}$ by arithmetic and harmonic means of previously found approximations. Let α_1 be any approximation of $\sqrt{a}$ such that $\alpha_1 > \sqrt{a}$. Then $\beta_1 = \frac{a}{\alpha_1}$ is also an approximation of $\sqrt{a}$ but deviates from the true value in the opposite direction because it follows from $\alpha_1 > \sqrt{a}$ that $\beta_1 = \frac{a}{\alpha_1} < \sqrt{a}$. We now derive a new pair of approximations, α_2 and β_2, by

$$\alpha_2 = \frac{\alpha_1 + \beta_1}{2} \qquad \beta_2 = \frac{a}{\alpha_2}$$

and continue this process by computing

$$\alpha_3 = \frac{\alpha_2 + \beta_2}{2} \qquad \beta_3 = \frac{a}{\alpha_3}$$

etc.[125f] It is evident that all the α's are greater than $\sqrt{a}$, all β's less than $\sqrt{a}$, but that each step diminishes the difference between corresponding approximations. We apply this method to $\sqrt{2}$ by starting with $\alpha_1 = \frac{3}{2}$ as the first rough approximation (α_1^2 would be 2;15). Then we obtain for the first pair

$$\alpha_1 = 1;30 \qquad \beta_1 = \frac{2}{1;30} = 1;20.$$

The next step leads to

$$\alpha_2 = \tfrac{1}{2}(1;30 + 1;20) = 1;25$$

$$\beta_2 = \frac{2}{1;25} = 1;24,42,21,\ldots..$$

Here we have already reached the above mentioned value $\sqrt{2} \approx 1;25$. The very next step leads to

$$\alpha_3 = \tfrac{1}{2}(1;25 + 1;24,42,21,\ldots.) = 1;24,51,10,\ldots.$$

the value given in (1).[125g] The fact that both values for $\sqrt{2}$ found in our texts are links of the same chain seems to be a rather strong argument in support of our explanation.[125h]

[125c] The reverse contains the figure of a rectangle with an inscribed diagonal, but the numbers are too badly preserved to warrant a restoration of the dimensions.

[125d] MKT I p. 104; cf. also MKT I p. 140.

[125e] VAT 6598; cf. MKT I pp. 279ff. and Neugebauer, Vorlesungen pp. 33ff.

[125f] From $\alpha_2 = \frac{1}{2}(\alpha_1 + \beta_1)$ and $\beta_2 = \frac{a}{\alpha_2}$ it follows that $\beta_2 = \frac{2a}{\alpha_1 + \beta_1} = \frac{2\alpha_1\beta_1}{\alpha_1 + \beta_1}$. This expression is known as the "harmonic mean" of α_1 and β_1.

[125g] More accurately: $\alpha_3 = 1;24,51,10,35,\ldots..$

[125h] It should be remarked that the expansion of $\sqrt{2}$ into a continued fraction also leads to (1), but not until the seventh step.

b. Trapezoids

The figure given on the obverse of **YBC 7290**[125i]

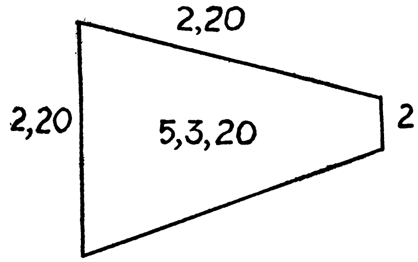


FIG. 4.

indicates that the area is obtained by

$$5;3,20 = 2;20 \cdot \frac{2;20 + 2}{2}.$$

On the reverse is given a trapezoid without inscribed numbers.

YBC 11126 is uninscribed on the reverse. The obverse gives the figure of a trapezoid with numbers.

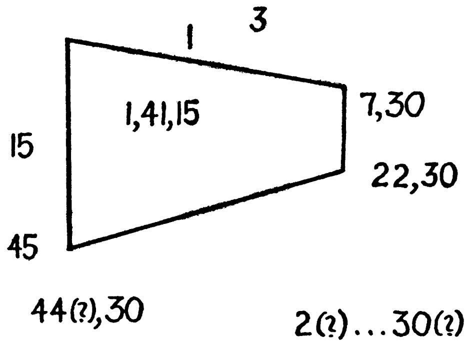


FIG. 5.

One set of these numbers satisfies the following relation

$$1,41,15 = 3,0 \cdot \frac{45 + 22;30}{2}$$

as expected for the area and sides. As for the remaining numbers, it is obvious that

$$45 = 3 \cdot 15 \qquad 22;30 = 3 \cdot 7;30 \qquad 3,0 = 3 \cdot 1,0$$

but the meaning of the coefficient 3 as well as the number 44(?),30 is not clear.

c. Circle

YBC 7302[125j] gives the figure of a circle whose circumference $c = 3$. We therefore have $c^2 = 9$ and for the

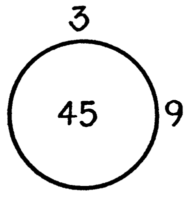


FIG. 6.

area

$$a = \frac{c^2}{4\pi} \approx \frac{c^2}{12} = 0;5 \cdot 9 = 0;45$$

using the value $\pi \approx 3$.[125k]

Analogously, we find in **YBC 11120**[125l]

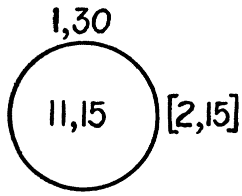


FIG. 7.

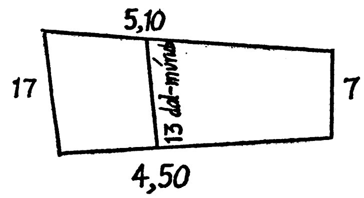
$$c = 1;30 \qquad c^2 = 2;15 \qquad a = 0;5 \cdot 2;15 = 0;11,15.$$

B and C. YBC 4675 and YBC 9852

Transcription of B (YBC 4675)

(Photograph: Plate 26; copy: Plate 1)

Obverse



1 *šum-ma* a(?)-šà(?)[126] uš uš *i* kú uš-1-e 5,10 uš-2-e 4,50
2 sag-an-ta 17 sag-ki-ta 7 a-šà-bi 2(bùr)$^{\text{iku}}$
3 1(bùr)$^{\text{iku}}$-ta-àm a-šà-*lam a-na ši-na zu-ú-uz*[127] *ta-al-li* *qá-ab-lu-ú ki ma-ṣi*

[125i] The tablet measures 7 by 8 cm.

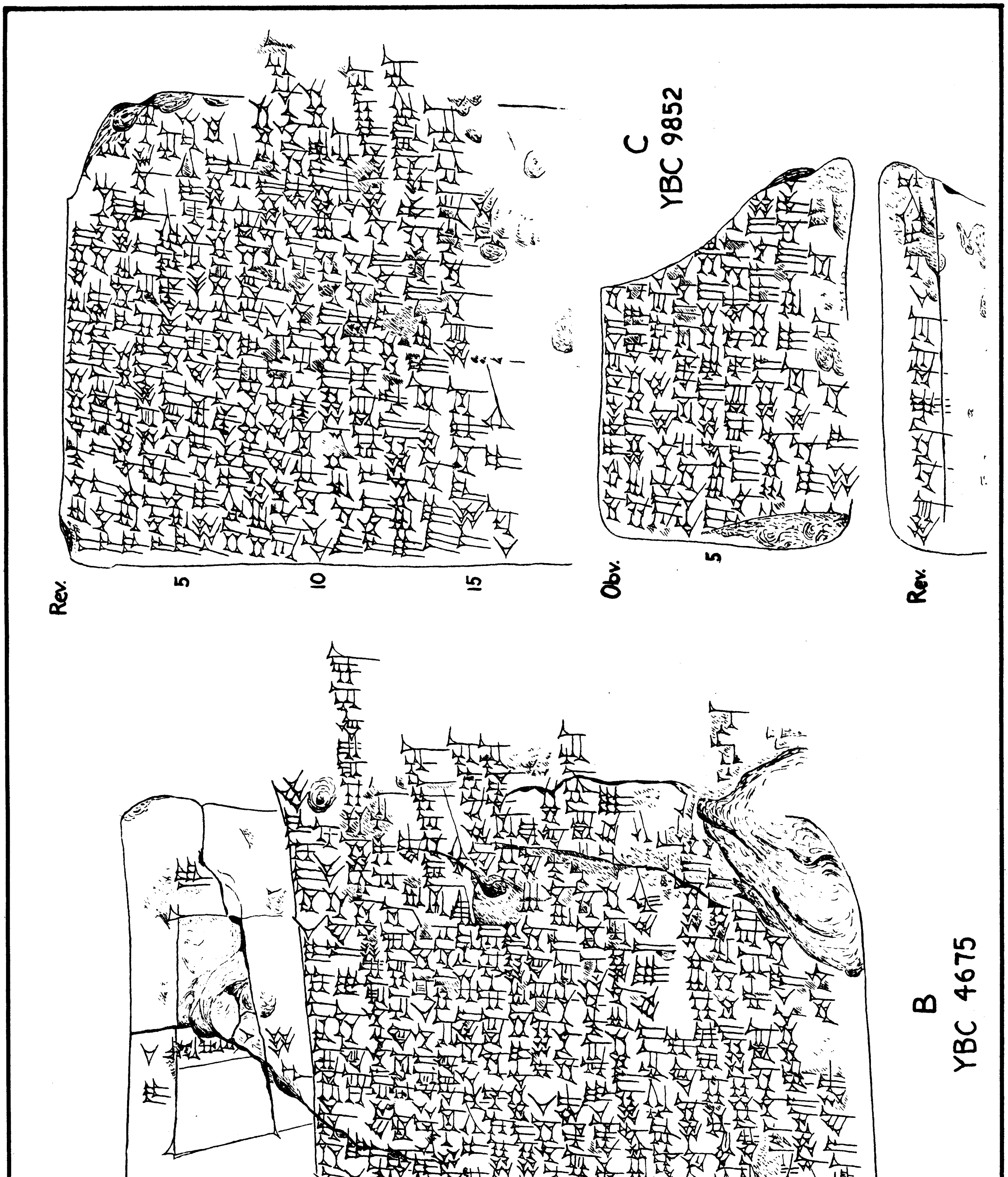
[125j] The shape of the tablet is roughly circular; diameter 8 cm.; the reverse is uninscribed.

[125k] Cf. above p. 9.

[125l] Width 8 cm.; length 6.5 cm. The figure appears on the reverse; the obverse is uninscribed.

[126] The reading of "a" in a(?)-šà(?) is doubtful because of a horizontal wedge and the low position of the lower wedge at the end; šà is suspicious because of the uneven arrangement of the diagonal wedges (contrast, e.g., line 20) and perhaps a fourth diagonal wedge above. The reading of the signs which follow, uš uš *i*, is based on a final cleaning of this line by Dr. A. Goetze after our copy of the text was prepared.

[127] Or should one emend to <*a-*>*zu-ú-uz* and translate "I divided"?


Obv.
Rev.
B
YBC 4675
C
YBC 9852
Obv.
Rev.

B

4 uš-gíd-da *ù* uš-lugúd-da *ki ma-ṣi lu-uš-ku-un-ma*
5 1(bùr)^{iku} *lu-ú sà-ni-iq ù a-na* 1(bùr)^{iku} *ša-ni-i-im*
6 *ki ma-ṣi* uš-gíd-da *ù ki ma-ṣi* uš-lugúd-da *lu-uš-ku-un-ma*
7 1(bùr)^{iku} *lu-ú sà-ni-iq* uš-ḫá *ga-me-ru-ú-tim*
8 *ki-la-a-al-le-e-en ta-ka-mar-ma ba-a-*[*ši-n*]*a te-ḫe-pe-e-ma*
9 5 *i-il-li-a-kum* igi 5 *ša i-li-a-ku*[*m t*]*a-pa-ṭa-ar-ma*
10 *a-na* sag-an-na *ša* 10 *e-li* sag-ki-ta *i-te-ru*
11 *a-na* 10 *ụa-at-ri-im ta-na-aš-ši-ma* 2 *i-na-an-di-kum*
12 *ta-as-sà-ḫa-ar* 17 sag-an-na *tu-uš-ta-ak-ka-al-ma*
13 4,49 *i-il-li-a-kum i-na li-ib-bi* 4,49
14 2 *ta-ḫa-ar-ra-aṣ-ma* 2,49 *a-ḫe-er-*[*t*]*um*
15 íb-si$_8$-*šu te-le-qé-e-ma*
16 13 *ta-al-lum qá-ab-lu-ú-um i-il-li-a-kum*
17 13 *ta-al-lam qá-ab-li-a-am ša i-li-a-*[*kum*]
18 *ù* 17 sag-an-na *ta-ka-mar-ma ba-a-*[*ši-na te-ḫe-pe*]*-e-ma*
19 15 *i-il-li-a-ak-kum* igi 15 *ta-p*[*a-ṭa-ar-ma*]
20 *a-na* 1(bùr)^{iku} a-šà-*im ta-na-*[*aš-ši-m*]*a*

Reverse

1 2 *i-na-an-di-kum* 2 *ša i-li-a-k*[*um*]
2 *a-na* 2 *a-ra-ka-re-e-em ta-na-aš-ši-m*[*a*]
3 4 *i-il-li-a-kum* 4 *ša i-li-a-k*[*um*]
4 *a-na* 2 UŠ *tu-uṣ-ṣa-ab-ma* 2,4 uš-gíd-d[a]
5 4 *i-na* 2 UŠ ki-2 *ta-ḫa-ar-ra-aṣ-ma*
6 1,56 uš-lugúd-da *te-ep-pe-eš-ma* 1(bùr)^{iku} *sà-ni-iq*
7 *ta-as-sà-ḫa-ar* 13 *ta-al-lam qá-ab-li-a-am*
8 *ša i-li-a-kum ù* 7 sag-ki-ta <*ta-*>*ka-mar*
9 *ba-a-ši-na te-ḫe-pe-e-ma* 10 *i-il-li-a-ak-kum*
10 igi 10 *ta-pa-ṭa-ar-ma a-na* 1(bùr)^{iku} a-šà-*im ta-na-aš-ši-ma*
11 3 UŠ *i-il-li-a-kum* 3 UŠ *ša i-li-a-ak-kum*
12 *a-na* 2 *a-ra-ka-re-e-em ta-na-aš-ši-ma*
13 6 *i-il-li-a-ak-kum* 6 *a-na* 3 UŠ *tu-uṣ-ṣa-ab-ma*
14 3,6 uš-gíd-da 6 *i-na* 3 UŠ *ta-ḫa-ar-ra-aṣ-ma*
15 2,54 uš-lugúd-da *tu-uš-ta-ak-ka-al-ma*
16 1(bùr)^{iku} *sà-ni-iq*

C (YBC 9852) is a duplicate of B (YBC 4675) reverse 7–16. Except for the last two lines, the distribution of the wording in the two texts is the same, line for line.

Transcription of C (YBC 9852)

(Photograph: Plate 29; copy: Plate 1)

Obverse

1 *ta-as-sà-ḫa-ar-ma* 13 *ta-*[*al-lam qá-ab-li-a-am*]
2 *ša i-li-a-kum ù* 7 sa[g-ki-ta *ta-ka-mar*]
3 *ba-a-ši-na te-ḫe-pe-e-ma* 10 [*i-il-li-a-ak-kum*]
4 igi 10 *ta-pa-ṭa-ar-ma a-na* 1(bùr)[^{iku} a-šà-*im ta-na-aš-ši-ma*]
5 3 *i-il-li-a-ak-kum* 3 *ša i-*[*li-a-ak-kum*]
6 *a-na* 2 *a-ra-ka-re-e-em ta-na-aš-*[*ši-ma*]
7 6 *i-il-li-a-kum* 6 *a-na* 3 *t*[*u-uṣ-ṣa-ab-ma*]
8 3,6 uš-gíd-da 6 *i-na* 3 *ta-ḫa-a*[*r-ra-aṣ-ma*]
9 2,54 uš-lugúd-da

Reverse

1 *tu-uš-ta-ak-ka-*<*al-*>*ma* 1(bùr)^{iku} *sà-ni-i*[*q*]

(Remainder of reverse uninscribed.)

Translation of B (YBC 4675)

Obverse

(For the figure see the transcription. Dal-múrub means "middle dividing-line.")
1 If . . . . The first length is 5,10, the second length 4,50,
2 the upper width 17, the lower width 7, its area 2 bùr;
3 divide[127] the area in two, each (part) 1 bùr. How large is my middle dividing-line?
4 How much should I set the longer length and the shorter length so that
5 1 bùr should border (on one side of the dividing-line), and for the second 1 bùr
6 how much should I set the longer length and how much the shorter length so that
7–8 1 bùr should border (on the other side)? Both the complete lengths you shall add together, then you shall halve [the]m, and
9 5,0 will result for you. You shall take the reciprocal of 5,0 which resulted for you, and then
10–11 you shall multiply (the result) with the excess 10 by which the upper width exceeded the lower width, and you will get 2,0.[128]
12 You shall turn about; you shall square 17, the upper width, and
13 4,49 will result for you. From 4,49
14 you shall subtract 2,0, and (as for) the remaining 2,49,
15 you shall take its square root, and
16 13, the middle dividing line, will result for you.
17–18 You shall add 13, the middle dividing-line which resulted for you, and 17, the upper width, then you shall hal[ve them], and
19 15 will result for you. You shall [take] the reciprocal of 15, [and]
20 you shall mult[iply (it)] by 1 bùr, the area, and

[128] Our translation of lines 10–11 deliberately departs from the awkward wording of the text: "to (= as for?) the upper width which exceeded the lower width by 10, you shall multiply (the result) with the excess 10, and you will get 2,0."

Reverse

[1]you will get 2,0. The 2,0 which resulted for you
[2]you shall multiply by 0;2, the *arakarūm*[129], and
[3]4 will result for you. The 4 which resulted for you
[4]you shall add to 2 UŠ, and the (resulting) 2,4 is the longer length.
[5]You shall subtract 4 from the second 2 UŠ, and
[6]the (resulting) 1,56 is the shorter length. You shall perform the (required) procedure, and the (resulting) 1 bùr borders (on the one side).
[7]You shall turn about; 13, the middle dividing-line
[8]which resulted for you, and 7, the lower width, you shall add together.
[9]You shall halve them, and 10 will result for you;
[10]you shall take the reciprocal of 10, and you shall multiply (it) by 1 bùr, the area, and
[11]3 UŠ will result for you. The 3 UŠ which resulted for you
[12]you shall multiply by 0;2, the *arakarūm*[129], and
[13]6 will result for you. You shall add 6 to 3 UŠ, and
[14]the (resulting) 3,6 is the longer length. You shall subtract 6 from 3 UŠ, and
[15]the (resulting) 2,54 is the shorter length. You shall make the (required) multiplication, and
[16]the (resulting) 1 bùr borders on the other side.

Commentary

a. Mathematical Commentary

The problem involves a trapezoid of given dimensions b_1, b_2, l_1, l_2 and area A to be divided into two parts with equal area (cf. the figure given in the transcription and fig. 8). The length d of the bisecting

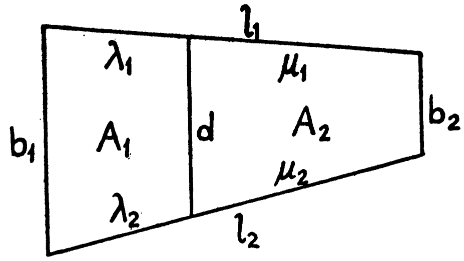


FIG. 8.

line and the resulting partial lengths of l_1 and l_2 are to be calculated. From the procedure followed by the text, it follows that b_1, b_2 and d are assumed to be parallel.

The given numerical values are

$$(1)\quad \begin{matrix} b_1 = 17\text{ GAR} & l_1 = 5{,}10\text{ GAR} \\ b_2 = 7\text{ GAR} & l_2 = 4{,}50\text{ GAR} \end{matrix} \quad A_1 = A_2 = 30{,}0\text{ GAR}^2.$$

[129] Cf. below p. 48.

In the text, A_1 and A_2 are expressed as 1 bùr each, which is equivalent[130] to 30,0 GAR²; in the case of the other four values, GAR is not given by the text but must be supplied. Because $l_1 = 5{,}10$ is about 18 times the width $b_1 = 17$, the trapezoid is a long strip. The assumption that one should read $l_1 = 5;10$ instead of 5,10 and correspondingly $l_2 = 4;50$ is excluded by the fact that we would have

$$l_1 + b_2 + l_2 = 5;10 + 7 + 4;50 = 17 = b_1,$$

an equation which would indicate the degeneration of the trapezoid into a single line of length b_1 (fig. 9).

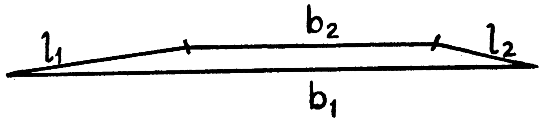


FIG. 9.

We must therefore assume the stretched form of the field as expressed by the numbers (1). This also follows from the units which are expressly indicated for the areas[131] and the terminology.[132] The scribe scarcely realized, however, that the numbers given are incompatible with the basic assumption of his calculations that b_1, b_2 and d are parallel lines. From the parallelism of b_1 and b_2, it would follow that one could form a triangle of sides $b_1 - b_2$, l_1 and l_2 (cf. fig. 10). But we known from (1) that

$$5{,}10 = l_1 \qquad l_2 + (b_1 - b_2) = 4{,}50 + 10 = 5{,}0$$

which shows the impossibility of such a triangle. In

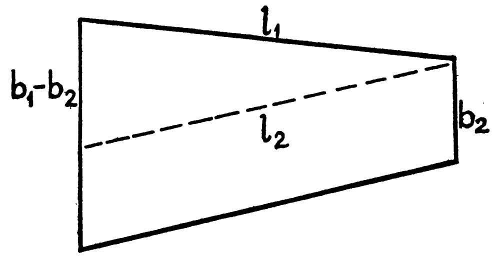


FIG. 10.

other words, the given dimensions exclude an arrangement of the sort indicated in fig. 8 and assumed by the calculation.[133] The formulas used are accordingly only of an *approximative* character.

[130] Cf. above p. 5.
[131] 1 bùr = 30,0 GAR², not 30 GAR².
[132] The "length" is never shorter than the "width".
[133] It can be shown by similar arguments that no angle can be 90°, but that l_1 can be assumed parallel to l_2; this, however, would not make the calculations of the text exact.

It follows from (1) that the area A of the figure is computed according to the approximative formula

$$(2) \qquad A = \frac{b_1 + b_2}{2} \cdot \frac{l_1 + l_2}{2},$$

and the calculations which follow are based on the corresponding formulas for the partial areas

$$(3) \quad A_1 = \frac{b_1 + d}{2} \cdot \frac{\lambda_1 + \lambda_2}{2} \qquad A_2 = \frac{d + b_2}{2} \cdot \frac{\mu_1 + \mu_2}{2}.$$

Before turning to the single steps of the calculation, we must derive still another relation used in the text.

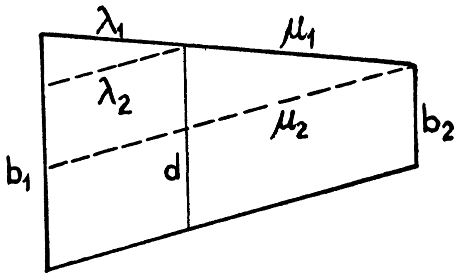


FIG. 11.

Assuming an arrangement like that indicated in fig. 8, we obtain (cf. fig. 11)

$$\frac{\lambda_1}{b_1 - d} = \frac{l_1}{b_1 - b_2} \qquad \frac{\lambda_2}{b_1 - d} = \frac{l_2}{b_1 - b_2}$$

and therefore

$$\lambda_1 + \lambda_2 = (b_1 - d)\frac{l_1 + l_2}{b_1 - b_2}.$$

Analogously,

$$\mu_1 + \mu_2 = (d - b_2)\frac{l_1 + l_2}{b_1 - b_2}.$$

Now, using the given equality $A_1 = A_2$ and (3), we obtain

$$(b_1 + d)(\lambda_1 + \lambda_2) = (d + b_2)(\mu_1 + \mu_2)$$

and from the two preceding equations by multiplying the first by $b_1 + d$, the second by $d + b_2$, we find that

$$(b_1 + d)(b_1 - d) = (d + b_2)(d - b_2)$$

or

$$b_1^2 - d^2 = d^2 - b_2^2$$

or finally

$$(4) \qquad d^2 = \tfrac{1}{2}(b_1^2 + b_2^2).$$

To repeat, this formula is derived under the assumption of the approximative formulas (3) and of the parallelism of b_1, b_2 and d.

We are now in a position to describe the procedure followed in the text, which is actually based on (4), albeit in a rather awkward form. Instead of calculating d directly from (4) and (1), the text employs a method which starts out as if only $b_1 - b_2$ were known but which thereafter makes use of the numerical value of b_2. The first step is

$$\tfrac{1}{2} \cdot 10,0 = 5,0 = \tfrac{1}{2}(l_1 + l_2).$$

The text now forms the reciprocal of this 5,0 but fails to mention a multiplication by 1,0,0 which, in view of the fact that "zeros" cannot be expressed, would have no effect on the numbers which are written in the text:

$$\frac{1,0,0}{5,0} \cdot 10 = 2,0 = \frac{2A}{l_1 + l_2}(b_1 - b_2) = \tfrac{1}{2}(b_1^2 - b_2^2).$$

The necessity of the multiplication by $A = 1,0,0$ follows from the next step:

$$17^2 = 4,49 = b_1^2$$

$$4,49 - 2,0 = 2,49 = b_1^2 - \tfrac{1}{2}(b_1^2 - b_2^2) = \tfrac{1}{2}(b_1^2 + b_2^2)$$

which assumes the correct order of magnitude for the "2". In accordance with (4), we have

$$d = \sqrt{\tfrac{1}{2}(b_1^2 + b_2^2)} = \sqrt{2,49} = 13,$$

which is the goal of the first part of the calculation.

The remaining part of the text is devoted to the determination of the sections λ_1, μ_1 of l_1 and λ_2, μ_2 of l_2. The main idea followed consists in computing both the sum and the difference of the λ's and μ's from which their single values follow immediately. To this end the text computes

$$\frac{13 + 17}{2} = 15 = \frac{d + b_1}{2}$$

and, using (3),

$$\overline{15} \cdot 30,0 = 2,0 = \frac{2A_1}{d + b_1} = \frac{\lambda_1 + \lambda_2}{2}$$

In order also to find $\frac{\lambda_1 - \lambda_2}{2}$, the text uses the proportion

$$\frac{\lambda_1}{\lambda_2} = \frac{l_1}{l_2}$$

which follows from fig. 8, assuming that b_1, d and b_2 are parallel. We therefore have

$$2,0 \cdot 0;2 = \frac{\lambda_1 + \lambda_2}{2} \cdot \frac{l_1 - l_2}{l_1 + l_2} = \frac{\lambda_1 - \lambda_2}{2}$$

and finally

$$(5) \quad 2,0 \pm 4 = \begin{Bmatrix} 2,4 \\ 1,56 \end{Bmatrix} = \frac{\lambda_1 + \lambda_2}{2} \pm \frac{\lambda_1 - \lambda_2}{2} = \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

which yields the points of intersection of the dividing line with the two longer sides.

We would now calculate the μ's by

$$\mu_1 = l_1 - \lambda_1 \qquad \mu_2 = l_2 - \lambda_2,$$

but the text prefers to repeat the same process for the second area as well. The single steps are

$$\frac{13+7}{2} = 10 = \frac{d+b_2}{2}$$

$$\overline{10} \cdot 30,0 = 3,0 = \frac{2A_2}{d+b_2} = \frac{\mu_1+\mu_2}{2}$$

$$3,0 \cdot 0;2 = 6 = \frac{\mu_1+\mu_2}{2} \cdot \frac{l_1-l_2}{l_1+l_2} = \frac{\mu_1-\mu_2}{2}$$

hence

$$(6) \quad 3,0 \pm 6 = \begin{Bmatrix} 3,6 \\ 2,54 \end{Bmatrix} = \frac{\mu_1+\mu_2}{2} \pm \frac{\mu_1-\mu_2}{2} = \begin{cases} \mu_1 \\ \mu_2 \end{cases}$$

which furnishes the remaining parts of the longer sides.

b. Terminology

1. *arakarūm*

The coefficient $\frac{l_1-l_2}{l_1+l_2}$ which is used to transform $\frac{\lambda_1+\lambda_2}{2}$ and $\frac{\mu_1+\mu_2}{2}$ into $\frac{\lambda_1-\lambda_2}{2}$ and $\frac{\mu_1-\mu_2}{2}$, respectively, is called *arakarūm*. For a discussion of the term, see p. 15.

2. *tallu* ≈ dal

The Akkadian word which is spelled *ta-al-li* (obv. 3), *ta-al-lum* (obv. 16) and *ta-al-lam* (obv. 17, rev. 7) proves definitely that the Sumerian word from which it was borrowed is to be read dal, not ri.[134] It also proves that *pirkum*[135] is not the unique Akkadian equivalent of Sumerian dal with the meaning indicated in the mathematical contexts. The translation "dividing-line" is intended to cover the technical meaning of dal ≈ *tallum*, *pirkum*, namely, the line which cuts through any area (e.g., circle, trapezoid, triangle).

134 For previous occurrences of dal in mathematical texts, see MKT II p. 31a under RI. For the use of dal as the altitude in a trapezoid, cf. below p. 50, No. 1.

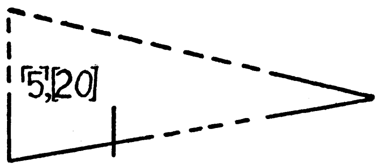
135 Cf. Neugebauer [1] p. 199. Outside the mathematical texts, *pirkum* has meanings such as "bar (of a door)" or "border (of a country)"; the meaning "width" is at present restricted to the Nuzi texts (cf. Lewy [1] p. 33).

Ca. MLC 1950

(Photograph: Plate 27; copy: Plate 2)

Transcription

Obverse



1 sag-dù 20 GAR uš
2 5,20 a-šà-bi
3 30 GAR . . . LU(?)

4 sag-an-ta *ù* sag-ki-ta *mi-nu-um* za-e kì-ta[-zu-dè]
5 igi 20 du₈ 3 *ta-mar* 3 *a-na* 5,20 *i-ši-ma* 1[6]
6 16 *a-na* sag-an-ta ù . . . - . . . -LU-*bu*
7 . . . - . . . -*ma* 30 uš *a-na* 2 *e-ṣí*(?)-*ip*[135a] 1
8 *ù* 20 *mu-ta-ri-tam* an-ta UL-gar 1,20
9 igi 1,20 du₈ 45 *a-na* 5,20 a-šà [*i-ši-ma* 4]
10 [4 *a-na*] 16 daḫ *i-na* 16 zi 2[0 sag-an-ta]
11 [12 sag-ki-ta]

Reverse destroyed except for traces at end.

Translation

Obverse

1 A triangle. 20 GAR is the length,
2 5,20 its area,
3 30 GAR the . . . .
4 What are the upper width and the lower width? [When you] perform (the operations),
5 take the reciprocal of 20, (and) you will see 0;3. Multiply 0;3 by 5,20, and (the result is) 1[6].
6 16 to(?) the upper width and . . . . .
7–8 . . . . 30, the length, multiply by 2, add (the resulting) 1,0 and 20, the upper perpendicular, (and the result is) 1,20.
9 Take the reciprocal of 1,20, [multiply] (the resulting) 0;0,45 by 5,20, the area, [and (the result is) 4].
10 Add [4 to] 16, subtract from 16. [The upper width is] 2[0];
11 [the lower width is 12.]

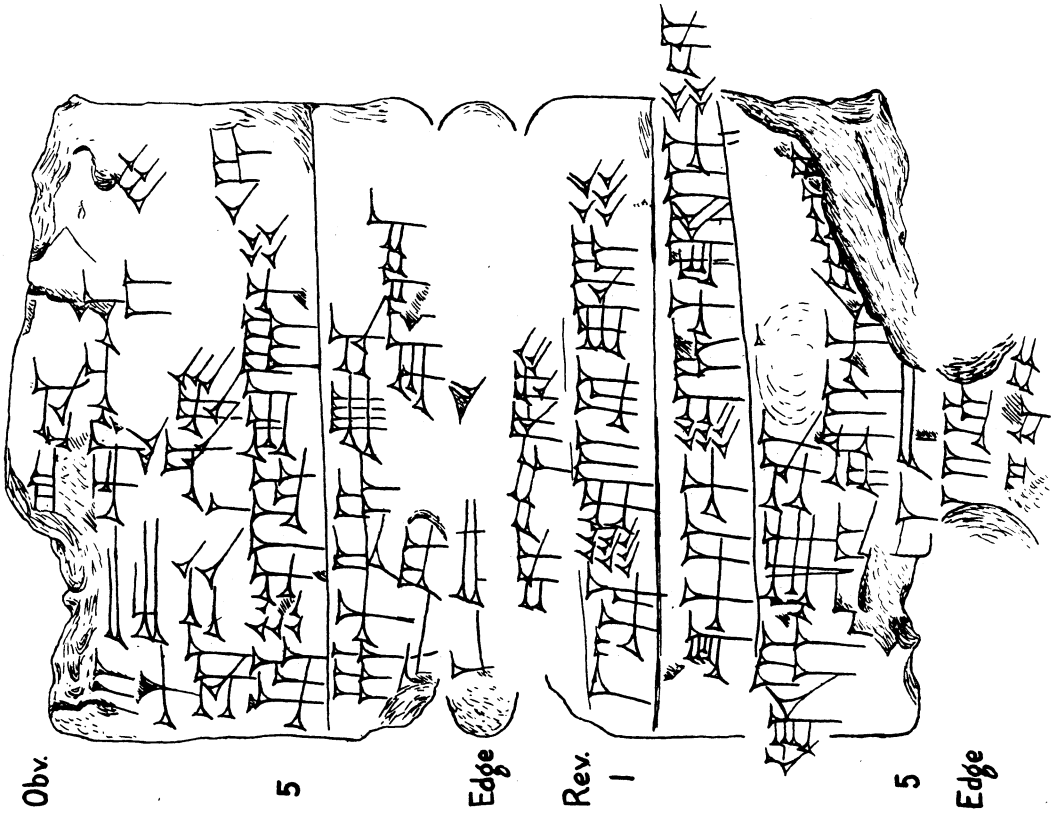
Reverse destroyed except for traces at the end.

Commentary

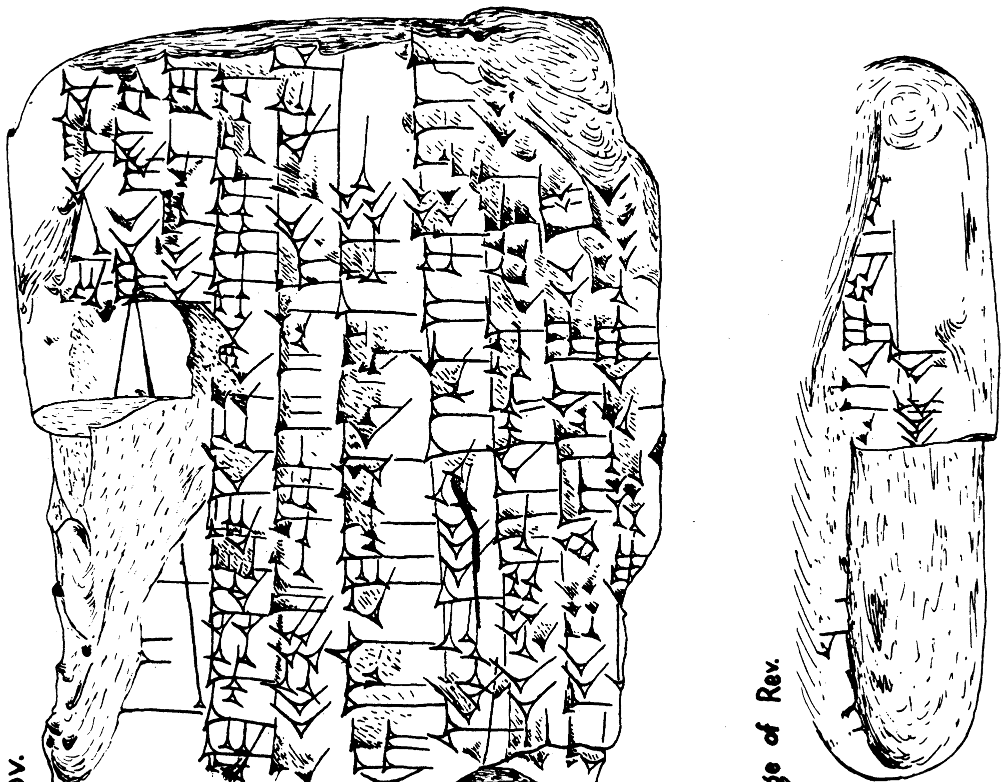
The problem presented here belongs to a class which is well attested:[135b] a triangle is subdivided by a line parallel to the base into a trapezoid and a smaller triangle. One group of dimensions is given; find the others from the given relations. In our particular case, we are given a triangle, the altitude 50, sub-

135a This reading, if correct, presupposes that the scribe inverted the order of the two components of the sign ZI.

135b Cf. the numerous examples cited in MKT III p. 82 under "Dreieck".



Ea. NBC 7934



Ca. MLC 1950

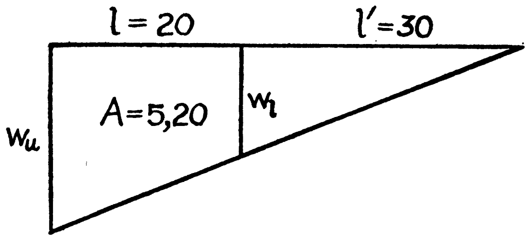


FIG. 12.

divided into two parts $l = 20$ and $l' = 30$.[135c] In addition, the area $A = 5{,}20$ of the trapezoid is given. Find the lengths w_u and w_l of the parallel sides of this trapezoid.[135d]

The method of solving the problem in question consists in finding both $\frac{1}{2}(w_u + w_l)$ and $\frac{1}{2}(w_u - w_l)$ such that the sum and difference of these expressions give the answer. The first expression is easily found. From the formula

(1) $$A = \tfrac{1}{2}(w_u + w_l)l$$

for the area of the trapezoid, it follows that we have

(2) $$\frac{A}{l} = \frac{5{,}20}{20} = 5{,}20 \cdot 0;3 = 16 = \tfrac{1}{2}(w_u + w_l),$$

as given in the text.

The next step requires explanation. We introduce

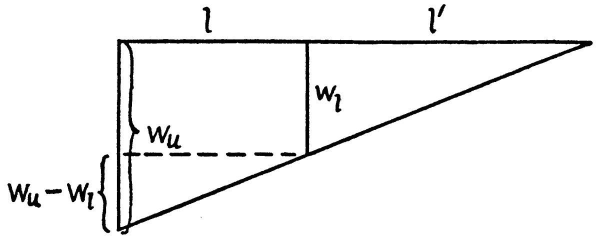


FIG. 13.

the dotted line as given in fig. 13 and then find, from similar triangles,

$$\frac{w_u}{l + l'} = \frac{w_l}{l'} = \frac{w_u - w_l}{l}$$

or

$$w_u = \frac{l + l'}{l}(w_u - w_l) \qquad w_l = \frac{l'}{l}(w_u - w_l).$$

By adding, we obtain

$$w_u + w_l = \frac{2l' + l}{l}(w_u - w_l)$$

and consequently

$$\tfrac{1}{2}(w_u - w_l) = \frac{(w_u + w_l)l}{2(2l' + l)}.$$

Using (1), we can therefore write

(3) $$\tfrac{1}{2}(w_u - w_l) = \frac{A}{2l' + l}$$

and this is precisely the relation used by the scribe. He computes:

$$2l' = 2 \cdot 30 = 1{,}0 \qquad 2l' + l = 1{,}0 + 20 = 1{,}20$$

$$\frac{1}{2l' + l} = 0;0{,}45$$

and therefore

(4) $$\frac{A}{2l' + l} = 0;0{,}45 \cdot 5{,}20 = 4 = \tfrac{1}{2}(w_u - w_l).$$

The relations (2) and (4) now provide the final answer

$$\tfrac{1}{2}(w_u + w_l) + \tfrac{1}{2}(w_u - w_l) = 16 + 4 = 20 = w_u$$

and

$$\tfrac{1}{2}(w_u + w_l) - \tfrac{1}{2}(w_u - w_l) = 16 - 4 = 12 = w_l.$$

Only parts of two lines of another problem are preserved on the reverse.

D. YBC 4608

(Photograph: Plate 28; copy: Plate 3)

Transcription

Obverse

1 ^{1}sag-ki-gu$_4$ [. . . . . .[136] a-]šà *a*[*-na ši-na zu-ú-uz*[137]]
^{2}42,11,15 a-šà íd-ki-t[a 14,3,45 a-šà íd-an-na]
^{3}igi-5-gál íd-ki-ta íd-a[n-na 52,30 dal-bi]
^{4}sa[g-an-n]a *ù* sag-ki-ta en[-nam] za-e k[ìd-da-zu-dè]
^{5}5 [*i-n*]*a* sag-ki-ta[139] ḫé-gar 1 [*i-*]*na* sag-an-na[138] ḫ[é-gar]
^{6}[1]4,3,45 *ù* 42,11,15 [gar-]gar-ma 56,15 in-sì
^{7}5 *ù* 1 gar-gar-ma 6 igi-6-gál-bi du$_8$-ma 10 in-sì
^{8}[10] *a-na* 56,15 nim-ma 9,22,30 *a-na ši-na* e-tab 18,45
^{9}[1]8,45 *re-eš-ka li-ki-il* igi-1-gál-bi *ša* sag-an-na[138] du$_8$-ma 1
^{10}1 *a-na* 14,3,45 nim-ma 14,3,45 in-sì *a-na ši-na* e-tab

[135c] For the sake of convenience, a right triangle is assumed in our commentary. It is equally possible to assume an isosceles triangle, each side 50. One would then also have to assume the approximative formula for the area of a trapezoid, which is found, e.g., in problem-text B (see p. 47, formula (2)).

[135d] The left side is, as usual, called the "upper width" because we must turn the figure 90° in clockwise direction (cf. p. 42, note 125b and Neugebauer, Vorlesungen, pp. 34 and 176).

[136] Probably blank.

[137] Restoration after problem-text B, obv. 3, q.v.

[138] Error for uš-an-na, "the upper length."

[139] Error for uš-ki-ta, "the lower length."